bounding the fields 11, etc., are determined from the values of p and u at the intersections of the corresponding Γ characteristics.

A further application of the weak shock approximation is shown in Fig. 13, where a common experimental problem is described in the (x, t) and (p, u) planes. A pressure-free flyer plate with uniform velocity w collides at $t = 0$ with a stationary pressure-free target, producing shock waves which travel forward into the target and backward into the flyer. The shock in the flyer reflects from the back face as a rarefaction, and there is subsequently a succession of reflections between free surface and interface which ultimately bring the flyer to a stop. The sequence of states, preserving continuity of u and p, is shown in the figure. The time between reflections is twice the travel time through the flyer, so the time to effectively stop the flyer can be estimated.

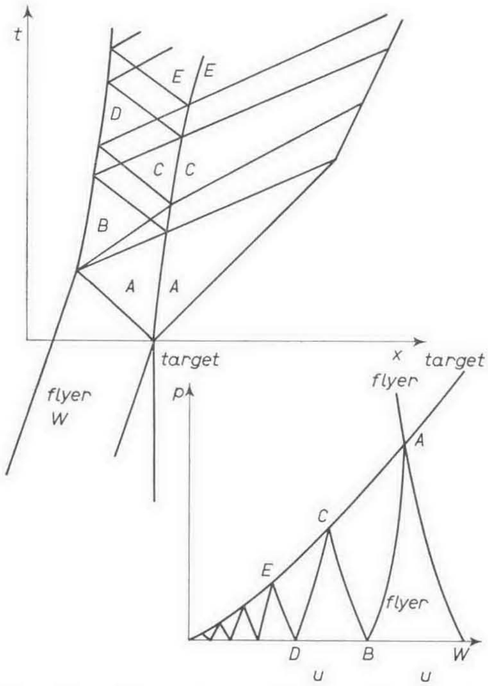

Fig. 13. – Flyer plate colliding with target.

4. – Elastic-plastic solids.

Much of the material in the preceding Sections has been couched in the language of fluids, though it applies equally well to solids, and has made but little reference to the explicit material properties involved. In this Section we become more specific about materials and examine more explicitly propagation effects in these models.

It must be recognized at the outset that there are no physically complete descriptions of the thermomechanical properties of solids. Hooke's law of elasticity is commonly used for small strains in metals and brittle solids, though there are materials to which it does not apply. Some materials are viscoelastic even at small strains, and the proper description of such materials is subject to current research. All solids fail through flow or fracture at some stress, and above this level, Hooke's law is totally improper. A satisfactory theory of fracture is far from realization; and the theory of plastic failure, while far advanced compared to fracture, is still logically incomplete and is often at odds with experimental results. However, the theory of plasticity is more completely formulated than other models of anelasticity, and its applications in shock propagation will be discussed here.

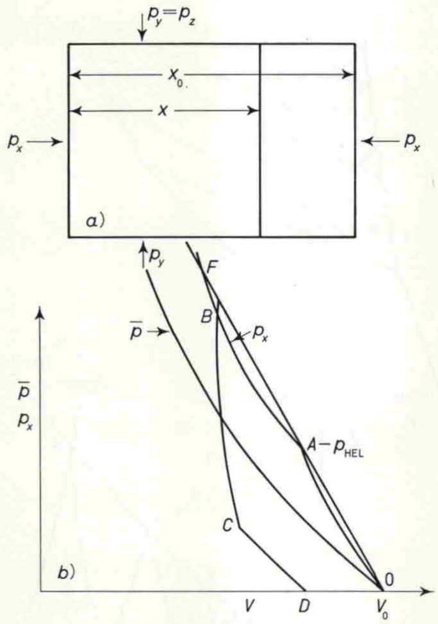

Fig. 14. – *a*) A parallelepiped of initial length x_0 has been compressed uniaxially to length x. *b*) Stress-strain relations for the sample of Fig. 14 *a*).

We restrict ourselves to the case of uniaxial strain existing in plane shock waves. We suppose a small element of volume to be compressed in the x-direction only and consider the relations between stress and strain. The notation used is shown in Fig. 14 *a*) and the expected stress-strain relation in a cycle of compression and rarefaction is shown in Fig. 14 *b*). Principal coordinates of the stress and strain matrices are (x, y, z) with x the direction of shock wave propagation. In order to maintain the condition of uniaxial strain while p_x is applied, p_y and p_z must be adjusted so as to maintain the lateral dimensions unchanged. Symmetry requires that $p_y = p_z$.

The most common assumptions of elasto-plasticity are:

i) Material response is elastic as long as deformation stresses do not exceed a characteristic value. The most commonly used criterion of failure is the von Mises condition

$$(p_x - p_y)^2 + (p_x - p_z)^2 + (p_y - p_z)^2 \leqslant 2Y^2 , \tag{41a}$$

where Y is the yield stress in simple tension. In uniaxial strain this becomes

$$|p_x - p_y| \leqslant Y . \tag{41b}$$

If the inequality applies, the material is elastic and satisfies Hooke's law:

$$p_x = \lambda\theta + 2\mu\varepsilon_x , \tag{42a}$$

$$p_y = \lambda\theta + 2\mu\varepsilon_y , \tag{42b}$$

$$p_z = \lambda\theta + 2\mu\varepsilon_z , \tag{42c}$$